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Radiative breaking scenario for the GUT gauge symmetry

T. Fukuyama[a], T. Kikuchi[b]

Department of Physics, Ritsumeikan University, Kusatsu, Shiga 525-8577, Japan

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Abstract. The origin of the grand unified theory (GUT) scale from the top-down perspective is explored. The GUT gauge symmetry is broken by the renormalization group effects, which is an extension of the radiative electroweak symmetry breaking scenario to the GUT models. That is, in the same way as the origin of the electroweak scale, the GUT scale is generated from the Planck scale through the radiative corrections to the soft supersymmetry breaking mass parameters. This mechanism is applied to a perturbative $SO(10)$ GUT model, recently proposed by us. In the $SO(10)$ model, the relation between the GUT scale and the Planck scale can naturally be realized by using order-one coupling constants.

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1 Introduction

A particularly attractive idea for the physics beyond the standard model (SM) is the possible appearance of grand unified theories (GUTs) [1]. The idea of a GUT bears several profound features. Perhaps the most obvious one is that GUTs have the potential to unify the diverse set of particle representations and parameters found in the SM into a single, comprehensive, and hopefully predictive framework. For example, through the GUT symmetry one might hope to explain the quantum numbers of the fermion spectrum, or even the origins of fermion mass. Moreover, by unifying all $U(1)$ generators within a non-Abelian theory, a GUT would also provide an explanation for the quantization of electric charge. By combining a GUT with supersymmetry (SUSY), we hope to unify the attractive features of a GUT simultaneously with those of SUSY into a single theory, SUSY GUT [2]. The apparent gauge coupling unification of the minimal supersymmetric standard model is strong circumstantial evidence in favor of the emergence of a SUSY GUT near $M_{\rm GUT} \simeq 2 \times 10^{16}$ [GeV] [3, 4].

While there are many appealing features in SUSY GUT, from a more fundamental theory point of view there is a problem in that there is a discrepancy between the fundamental scale, say the (reduced) Planck scale, $M_{\rm Pl} \simeq 2.4 \times 10^{18}$ [GeV], and the GUT scale, $M_{\rm GUT} \simeq 2 \times 10^{16}$ [GeV]. There have already been many approaches to this problem. Recent development of the extra-dimensional physics may provide one of the solutions [5]. That says that the discrepancy is a consequence of a distortion of the renormalization group (RG) runnings by the change of space dimensions, and the true GUT scale would be raised to the Planck scale. Though it is interesting, here we seek other approaches. That is, the 'dynamical' generation of the GUT scale. Namely, we assume that the theory has no dimensionful parameters at the beginning except for the Planck scale. The GUT scale, which we call it from the low-energy perspective, is generated from the radiative corrections. That lifts the flatness of the original potential. Then the GUT scale emerges from the dynamics. This has already been used to break the electroweak gauge symmetry [6–11] that may be regarded as the first evidence of some supersymmetric extensions of the standard model.

In this paper, we follow the same idea but extend the gauge group of the electroweak theory $SU(2)_L \times U(1)_Y$ to a simple gauge group for the GUT, e.g. $SO(10)$, and consider to break it to the standard model one, $SU(3)_c \times SU(2)_L \times U(1)_Y$ by radiative corrections. Indeed, the soft SUSY breaking mass parameters for the GUT Higgs multiplet can be driven to negative values at the GUT scale through their RG runnings [12–16]. We explicitly construct a model with such a radiative GUT breaking scenario, and apply it to the $SO(10)$ model that allows perturbative calculations up to the Planck scale and satisfies low-energy phenomena [17]. Then, the GUT scale is determined only by the order-one Yukawa couplings and the Planck scale. This deep connection between the GUT scale and the Planck scale leads us to believe the theory of grand unification.

2 Toy model

First we consider an $SU(5)$ GUT model discussed in [15], clarifying the argument. Let us denote S, H, $\overline{H}$, and Σ

[a] e-mail: fukuyama@se.ritsumei.ac.jp
[b] e-mail: tatsuru.kikuchi@gmail.com

as $SU(5)$ singlet, fundamental, anti-fundamental, and the GUT breaking adjoint Higgs superfields, respectively. By giving global $U(1)$ charges for these fields as $S=+2$, $H=\overline{H}=+\frac{1}{2}$, $\Sigma=-1$, we have a superpotential of the form

$$W(S,\Sigma)=\lambda_\Sigma S\mathrm{Tr}(\Sigma^2)+\lambda_H\overline{H}\Sigma H\,. \tag{1}$$

Obviously, from (1) we obtain

$$F_\Sigma^\dagger=\frac{\partial W}{\partial\Sigma}=2\lambda_\Sigma S\Sigma+\lambda_H\overline{H}H=0\,,$$
$$F_S^\dagger=\frac{\partial W}{\partial S}=\lambda_\Sigma\mathrm{Tr}(\Sigma^2)=0\,. \tag{2}$$

That leads to one of the vacua: $\langle\Sigma\rangle=0$, $\langle S\rangle=$ arbitrary, regarding the electroweak scale vacuum expectation values (VEVs) as zero: $\langle H\rangle=\langle\overline{H}\rangle=0$. For the true vacuum which respects the SM gauge symmetry $SU(3)_c\times SU(2)_L\times U(1)_Y$, we want to have a non-zero VEV for the adjoint Higgs field in the following direction[1]:

$$\langle\Sigma\rangle=\frac{1}{\sqrt{30}}\mathrm{diag}(2,2,2,-3,-3)\,v\,,\quad v\neq0\,. \tag{3}$$

Here we include the soft SUSY breaking mass terms,

$$V_{\mathrm{soft}}=m_\Sigma^2|\Sigma|^2+m_S^2|S|^2+\frac{1}{2}M_\lambda\lambda_a^{\mathrm{T}}C^{-1}\lambda_a\,, \tag{4}$$

where $\lambda_a(a=1,\ldots,24)$ is the $SU(5)$ gaugino. Taking this into account, the total scalar potential is given by

$$V=m_\Sigma^2|\Sigma|^2+m_S^2|S|^2+|2\lambda_\Sigma S\Sigma|^2+\left|\lambda_\Sigma\mathrm{Tr}(\Sigma^2)\right|^2$$
$$+\frac{1}{2}M_\lambda\lambda_a^{\mathrm{T}}C^{-1}\lambda_a\,. \tag{5}$$

Then the potential minima can be obtained as follows:

$$\frac{\partial V}{\partial\Sigma^\dagger}=\left(m_\Sigma^2+4\lambda_\Sigma^2S^2\right)\Sigma=0\,, \tag{6}$$
$$\frac{\partial V}{\partial S^\dagger}=\left(m_S^2+4\lambda_\Sigma^2\mathrm{Tr}(\Sigma^2)\right)S=0\,, \tag{7}$$

that is, one of the vacua which respects the SM gauge symmetry is found to be

$$\langle S\rangle\simeq\sqrt{\frac{-m_\Sigma^2}{4\lambda_\Sigma^2}}\,,$$
$$\langle\Sigma\rangle=\frac{1}{\sqrt{30}}\mathrm{diag}(2,2,2,-3,-3)v\,,\quad v\simeq\sqrt{\frac{-m_S^2}{4\lambda_\Sigma^2}}\,. \tag{8}$$

Here the negative mass squared for the singlet, $m_S^2<0$, should be satisfied at the GUT scale to realize the correct symmetry breaking. In the following, we really see that such a negative value can be achieved through the RG running from the Planck scale to the GUT scale with a large enough Yukawa coupling even if we start with a positive mass squared at the Planck scale.

[1] Though there is an equivalent possibility to have a VEV in the other direction $SU(4)\times U(1)\subset SU(5)$, here we just take the VEV in the desirable SM direction by hand.

3 RG analysis

The RG equations for the Yukawa couplings and the soft SUSY breaking mass terms are given by [18, 19]

$$16\pi^2\mu\frac{\mathrm{d}\lambda_\Sigma}{\mathrm{d}\mu}=\left(14\lambda_\Sigma^2-20g^2\right)\lambda_\Sigma\,,$$
$$16\pi^2\mu\frac{\mathrm{d}m_S^2}{\mathrm{d}\mu}=24\lambda_\Sigma^2\left(m_S^2+2m_\Sigma^2\right)\,,$$
$$16\pi^2\mu\frac{\mathrm{d}m_\Sigma^2}{\mathrm{d}\mu}=2\lambda_\Sigma^2\left(m_S^2+2m_\Sigma^2\right)-40M_a^2\,, \tag{9}$$
$$16\pi^2\mu\frac{\mathrm{d}M_a}{\mathrm{d}\mu}=-20g^2M_a\,,$$
$$16\pi^2\mu\frac{\mathrm{d}g}{\mathrm{d}\mu}=-10g^3\,.$$

In the limit of the vanishing gaugino masses, that is, in the exact $\mathcal{R}$ symmetric limit, the RG equations for the soft SUSY breaking scalar masses lead to the following forms:

$$\mu\frac{\mathrm{d}m_S^2}{\mathrm{d}\mu}=\frac{3}{2\pi^2}\lambda_\Sigma^2\left(m_S^2+2m_\Sigma^2\right)\,,$$
$$\mu\frac{\mathrm{d}m_\Sigma^2}{\mathrm{d}\mu}=\frac{1}{8\pi^2}\lambda_\Sigma^2\left(m_S^2+2m_\Sigma^2\right)\,. \tag{10}$$

Here we have assumed the coupling constant λ_Σ being a constant number against the renormalization from M_{Pl} to M_{GUT}. One combination leads to the following equation:

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\left(m_S^2+2m_\Sigma^2\right)=\frac{7}{4\pi^2}\lambda_\Sigma^2\left(m_S^2+2m_\Sigma^2\right)\,. \tag{11}$$

Assuming the universal soft mass parameter $m_{3/2}$ at the Planck scale, the solution is found to be

$$m_S^2+2m_\Sigma^2=3m_{3/2}^2\exp\left[\frac{7}{4\pi^2}\lambda_\Sigma^2\ln\left(\frac{\mu}{M_{\mathrm{Pl}}}\right)\right]\,. \tag{12}$$

It gives a solution for m_S^2 as follows:

$$m_S^2=-\frac{11}{7}m_{3/2}^2+\frac{18}{7}m_{3/2}^2\exp\left[\frac{7}{4\pi^2}\lambda_\Sigma^2\ln\left(\frac{\mu}{M_{\mathrm{Pl}}}\right)\right]\,, \tag{13}$$

and also the solution for m_Σ^2 is found to be

$$m_\Sigma^2=\frac{11}{14}m_{3/2}^2+\frac{3}{14}m_{3/2}^2\exp\left[\frac{7}{4\pi^2}\lambda_\Sigma^2\ln\left(\frac{\mu}{M_{\mathrm{Pl}}}\right)\right]\,. \tag{14}$$

The solution for m_S^2 has two opposite-sign terms as one can see in (13), and the RG running from the Planck scale may drive it to the negative value to induce the GUT symmetry breaking. Here we show the relation explicitly. The required condition to achieve the radiative GUT symmetry breaking is $m_S^2=0$ at a scale $\mu=M_{\mathrm{GUT}}$. From this requirement, the GUT scale is generated from the Planck scale via the dimensional transmutation

$$M_{\mathrm{GUT}}=M_{\mathrm{Pl}}\exp\left[\frac{4\pi^2}{7\lambda_\Sigma^2}\ln\left(\frac{11}{18}\right)\right]\,. \tag{15}$$

One of the important things is that this expression depicting the GUT scale is completely independent of the SUSY breaking scale $m_{3/2}$, and the order-one coupling constant ($\lambda_\Sigma \simeq 0.72$) can realize the appropriate GUT scale $M_{\rm GUT} \simeq 2\times 10^{16}$ [GeV]. This completely reflects the situation similar to the radiative electroweak symmetry breaking scenario [6–11]. This scenario postulates that the electroweak scale ($v \simeq 174$ [GeV]) is a consequence of the large top Yukawa coupling ($y_t \simeq m_t/v \simeq 1.02$), which makes one of the soft SUSY breaking mass parameters for the Higgs doublets (H_u, H_d) bend to the negative value ($m_{H_u}^2 < 0$ at $\mu = M_{\rm EW}$) through the RG running from the Planck scale at which the soft SUSY breaking mass parameters are assumed to be positive ($m_{H_u}^2 > 0$ at $\mu = M_{\rm Pl}$).

4 $SO(10)$ model

Now we proceed to extend the model into the realistic $SO(10)$ GUT model, in which the adjoint representation is necessary to break the $SO(10)$ and to provide the appropriate numbers of the would-be Nambu–Goldstone (NG) boson. For details of symmetry breaking patterns in $SO(10)$ models, see [21]. The use of the $A = \mathbf{45}$ representation of the Higgs field is also economical for the realization of the doublet–triplet splittings in the $SO(10)$ GUT with the help of the Dimopoulos–Wilczek mechanism [22]. From (15), the coupling constant λ_A is given by $\lambda_A \simeq 0.72$, which is a natural number to realize in going through the perturbative calculation [17]. In [17], we introduced a set of the Higgs as $\{\mathbf{10}+\mathbf{10}'+\mathbf{45}+\mathbf{16}+\overline{\mathbf{16}}\}$ that is denoted by $H = \mathbf{10}$, $H' = \mathbf{10}'$, $A = \mathbf{45}$, $\psi = \mathbf{16}$, and $\overline{\psi} = \overline{\mathbf{16}}$. The Yukawa couplings with matter multiplet $\Psi_i = \mathbf{16}_i$ ($i = 1,2,3$) are given by

$$W = Y_{10}^{ij}\Psi_i\Psi_j H + \frac{1}{M_{\rm Pl}} Y_{45}^{ij}\Psi_i\Psi_j H' A + \frac{1}{M_{\rm Pl}} Y_{16}^{ij}\Psi_i\Psi_j \overline{\psi}\overline{\psi}\,. \tag{16}$$

The first two terms are the Yukawa couplings of quarks, charged leptons, and Dirac neutrinos. The third term is that for heavy right-handed Majorana neutrinos which make light Majorana neutrinos via the see-saw mechanism [23]. This is a minimal set of the Higgs which realizes the realistic fermion mass spectra and achieves the correct gauge symmetry breaking. In addition to it, we add one singlet, $S = \mathbf{1}$, and one $\mathbf{54}$ multiplet, $S' = \mathbf{54}$. Assuming global $U(1)$ charges $\Psi_i = -1$ ($i = 1,2,3$), $S = -2$, $S' = +2$, $A = -1$, $\psi = +1$, $\overline{\psi} = +1$, $H = +2$, and $H' = +3$ for these fields, the relevant part of the superpotential for the GUT breaking sector is given by

$$W = \lambda_\psi S\overline{\psi}\psi + \lambda_A S' A^2\,, \tag{17}$$

and the corresponding soft mass terms are

$$\begin{aligned} V_{\rm soft} = {}& m_H^2|H|^2 + m_{H'}^2|H'|^2 + m_{\overline{\psi}}^2|\overline{\psi}|^2 + m_\psi^2|\psi|^2 \\ &+ m_A^2|A|^2 + m_S^2|S|^2 + m_{S'}^2|S'|^2 \\ &+ \frac{1}{2}M_\lambda \lambda_a^{\rm T} C^{-1}\lambda_a\,. \end{aligned} \tag{18}$$

These give a total scalar potential as follows:

$$\begin{aligned} V = {}& m_H^2|H|^2 + m_{H'}^2|H'|^2 + m_{\overline{\psi}}^2|\overline{\psi}|^2 + m_\psi^2|\psi|^2 \\ &+ \left|\lambda_\psi S\overline{\psi}\right|^2 + |\lambda_\psi S\psi|^2 + \left|\lambda_\psi\overline{\psi}\psi\right|^2 \\ &+ m_A^2|A|^2 + m_S^2|S|^2 + m_{S'}^2|S'|^2 \\ &+ |2\lambda_A S' A|^2 + \left|\lambda_A\left(A^2 - \frac{1}{10}{\rm Tr}(A^2)\mathbf{1}\right)\right|^2 \\ &+ \frac{1}{2}M_\lambda\lambda_a^{\rm T}C^{-1}\lambda_a\,, \end{aligned} \tag{19}$$

and one of the vacua which respects the SM gauge symmetry is

$$\begin{aligned} &\langle H\rangle = \langle H'\rangle = 0\,, \quad \langle\psi\rangle = \langle\overline{\psi}\rangle \simeq \sqrt{\frac{-m_S^2}{\lambda_\psi^2}}\,, \\ &\langle S\rangle \simeq \sqrt{\frac{-m_\psi^2}{\lambda_\psi^2}}\,, \\ &\langle S'\rangle = \frac{1}{\sqrt{60}}\begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes{\rm diag}(2,2,2,-3,-3)s'\,, \\ &s' \simeq \sqrt{\frac{-m_A^2}{4\lambda_A^2}}\,, \\ &\langle A\rangle = \frac{1}{\sqrt{10}}\begin{pmatrix}0&1\\-1&0\end{pmatrix}\otimes{\rm diag}(1,1,1,1,1)a\,, \\ &a \simeq \sqrt{\frac{-m_{S'}^2}{4\lambda_A^2}}\,. \end{aligned} \tag{20}$$

The RG equations for the soft SUSY breaking parameters (in the limit of vanishing gaugino masses) are given by [18, 19][2]

$$\begin{aligned} 16\pi^2\mu\frac{{\rm d}m_S^2}{{\rm d}\mu} &= 16\lambda_\psi^2\left(m_S^2 + 2m_\psi^2\right)\,, \\ 16\pi^2\mu\frac{{\rm d}m_\psi^2}{{\rm d}\mu} &= 2\lambda_\psi^2\left(m_S^2 + 2m_\psi^2\right)\,, \\ 16\pi\mu\frac{{\rm d}m_{S'}^2}{{\rm d}\mu} &= 45\lambda_A^2\left(m_{S'}^2 + 2m_A^2\right)\,, \\ 16\pi^2\mu\frac{{\rm d}m_A^2}{{\rm d}\mu} &= 54\lambda_A^2\left(m_{S'}^2 + 2m_A^2\right)\,. \end{aligned} \tag{21}$$

Then the solutions for m_S^2, m_ψ^2, $m_{S'}^2$, and m_A^2 are now found to be

$$m_S^2 = -\frac{7}{5}m_{3/2}^2 + \frac{12}{5}m_{3/2}^2\exp\left[\frac{5\lambda_\psi^2}{4\pi^2}\ln\left(\frac{\mu}{M_{\rm Pl}}\right)\right]\,, \tag{22}$$

$$m_\psi^2 = \frac{7}{10}m_{3/2}^2 + \frac{3}{10}m_{3/2}^2\exp\left[\frac{5\lambda_\psi^2}{4\pi^2}\ln\left(\frac{\mu}{M_{\rm Pl}}\right)\right]\,, \tag{23}$$

[2] In principle, these results can be read off from the results of [20] for a general gauge theory.

$$m_{S'}^2 = \frac{2}{17} m_{3/2}^2 + \frac{15}{17} m_{3/2}^2 \exp\left[\frac{153\lambda_A^2}{16\pi^2} \ln\left(\frac{\mu}{M_{\rm Pl}}\right)\right] , \tag{24}$$

$$m_A^2 = -\frac{1}{17} m_{3/2}^2 + \frac{18}{17} m_{3/2}^2 \exp\left[\frac{153\lambda_A^2}{16\pi^2} \ln\left(\frac{\mu}{M_{\rm Pl}}\right)\right] . \tag{25}$$

In the $SO(10)$ case, we have two opposite-sign terms for m_S^2 and m_A^2, and the RG runnings from the Planck scale drive both of them to the negative values to induce the GUT symmetry breaking but also to break the rank of $SO(10)$, which is necessary to realize the standard model gauge group. Here we recall the decompositions of each representation under the subgroups of $SO(10)$:

$$\begin{aligned} \mathbf{54} &= (\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{3},\mathbf{3}) + (\mathbf{20'},\mathbf{1},\mathbf{1}) + (\mathbf{6},\mathbf{2},\mathbf{2}) \\ &\quad \text{under } \mathsf{SU}(4)_c \times \mathsf{SU}(2)_L \times \mathsf{SU}(2)_R , \end{aligned} \tag{26}$$

$$\begin{aligned} \mathbf{16} &= (\mathbf{4},\mathbf{2},\mathbf{1}) + (\overline{\mathbf{4}},\mathbf{1},\mathbf{2}) \\ &\quad \text{under } \mathsf{SU}(4)_c \times \mathsf{SU}(2)_L \times \mathsf{SU}(2)_R \\ &= [(\mathbf{3},\mathbf{2},1/6) + (\mathbf{1},\mathbf{2},-1/2)] \\ &\quad + \left[(\overline{\mathbf{3}},\mathbf{1},1/3) + (\overline{\mathbf{3}},\mathbf{1},-2/3) + (\mathbf{1},\mathbf{1},1) + (\mathbf{1},\mathbf{1},0)\right] \\ &\quad \text{under } \mathsf{SU}(3)_c \times \mathsf{SU}(2)_L \times \mathsf{U}(1)_Y . \end{aligned} \tag{27}$$

Hence, if $(\mathbf{1},\mathbf{1},\mathbf{1}) \in \mathbf{54}$ develops a VEV, the gauge symmetry breaks down to $SU(4)_c \times SU(2)_L \times SU(2)_R$, and further if $(\mathbf{1},\mathbf{1},0) \in \mathbf{16}$ develops a VEV, the gauge symmetry breaks down to the standard model one.

Equation (21) shows that the RG effects for the coupled system of $\{S, \psi, \overline{\psi}\}$ are stronger than that of $\{S', A\}$. So, combined with the above discussion, this fact implies that $SO(10)$ first breaks to $SU(4)_c \times SU(2)_L \times SU(2)_R$ via $\langle S' \rangle$ and soon breaks to $SU(3)_c \times SU(2)_L \times U(1)_Y$ via $\langle \psi \rangle$, leading to the scenario that $SO(10)$ breaks to the MSSM at the GUT scale. It should be remarked that this breaking pattern does not much depend on the choice of the coupling constants for most of the parameter region because of their large charge difference between $\{\mathbf{1} \oplus 16 \oplus \overline{16}\}$ and $\{\mathbf{54} \oplus 45\}$. Note that the VEV of $\langle \psi \rangle$ breaks $B-L$ and gives masses to the heavy right-handed Majorana neutrinos at the same time. In summary, the GUT scale is completely determined only by the order-one coupling constants $\lambda_A \sim \lambda_\psi \sim 1$.

Finally, we determine the concrete values for these coupling constants. To achieve a simple, one-step unification picture, we impose the condition that the rank breaking occurs at the same time as the GUT breaking. Then the required conditions to achieve the radiative GUT symmetry breaking become $m_A^2 = 0$ and $m_S^2 = 0$ at a scale $\mu = M_{\rm GUT}$. From this requirement, the GUT scale is generated from the Planck scale via the dimensional transmutation as in the case of $SU(5)$:

$$\begin{aligned} M_{\rm GUT} &= M_{\rm Pl} \exp\left[\frac{16\pi^2}{153\lambda_A^2} \ln\left(\frac{1}{18}\right)\right] \\ &= M_{\rm Pl} \exp\left[\frac{4\pi^2}{5\lambda_\psi^2} \ln\left(\frac{7}{12}\right)\right] . \end{aligned} \tag{28}$$

From these equations, we can determine the values of order-one coupling constants which are necessary to realize the GUT scale of order $M_{\rm GUT} \simeq 2 \times 10^{16}$ [GeV] as $\lambda_A \simeq 0.79$ and $\lambda_\psi \simeq 0.94$.

5 Conclusion

In this letter, we have explored the origin of the GUT scale. Given the universal soft mass at the Planck scale, the GUT scale is determined in terms of the order-one coupling constants. That is, the positive soft mass runs from the Planck scale to the GUT scale according to the RG equations and crosses zero at the GUT scale, which is exactly the same idea as the radiative electroweak symmetry breaking scenario in the MSSM with a suitable boundary condition at the Planck scale. This mechanism has been applied to a $SO(10)$ model recently proposed by us [17], which is compatible with low-energy phenomena. In this $SO(10)$ model, the GUT scale can be generated from the Planck scale by using order-one coupling constants and the symmetry breaking pattern of $SO(10)$ is also specified. Thus, the GUT scale is determined both by a top-down (from the Planck scale to the GUT scale) scenario as well as a bottom-up (from the electroweak scale to the GUT scale) scenario, and they coincide with each other.

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